Iterative Decoding and Channel Estimation

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Abstract — **We investigate iterative decoding and channel estimation for multiple-access channels. Results are obtained concerning the fixed points of such iterations.**

I. Iterative Receiver Principle

In [1] an iterative receiver was proposed for the linear multiple access channel. We now consider an approach for integration of channel estimation into this technique whereby we use the a-posteriori probabilities of the information symbols as uncertain training sequences for the purposes of channel estimation. We investigate the properties of fixed points of such iterations.

Let $\mathcal{S}$ be a vector space. Unconstrained sequences can take any value $u \in \mathcal{S}$ as opposed to constrained sequences $x \in \mathcal{C} \subset \mathcal{S}$. We are interested in low-complexity joint detection (or estimation) for sets of constrained sequences observed according to known transition probabilities. These probabilities are defined by some combination of deterministic mappings (e.g. linear combining) and non-deterministic perturbations (e.g. noise).

Suppose that the sequences x_k, $k = 1, \ldots, n$ are each produced by a mapping $\mathcal{C}_k$ of an unconstrained sequence u_k. The random sequence y is observed according to $p(y \mid x_1, \ldots, x_n)$. The u_k may or may not be independent, but are conditionally dependent given y. This model can be thought of as a multiple-access communications system (the x_k are coded information sequences), but is rich enough to describe other systems of interest, such as inter-symbol interference channels (by allowing some of the x_k to represent the sequence of channel taps, obeying known spectral constraints) and space-time diversity channels.

Optimal detection means the determination of either the posterior density $p(u_1, u_2, \ldots, u_n \mid y)$, or its marginals, taking into account the constraints. This is usually an NP-complete problem and we propose a reduced complexity iterative algorithm. The basic principle that we propose for design of such algorithms may be stated concisely as follows.

1. Incorporate dependence, ignore constraints.
2. Incorporate constraints, ignore dependence.

We iteratively update the distributions $p_k(u_k)$. Ideally p_k converges over iteration to the k-th marginal of the true posterior distribution $p(u_1, u_2, \ldots, u_n \mid y)$. The principle also applies to estimation problems, in which case the distributions are replaced with the current estimates, which we hope converge to some desired estimator e.g. MMSE.

Let $p = \{p_1(u_1), p_2(u_2), \ldots, p_n(u_n)\}$ be the sequence priors. At the conclusion of any iteration step, the *unconstrained joint detector*, using as priors the current set of marginal distributions p, produces a new set p^+, taking into account only the conditional dependencies. All the constraints are relaxed. This results in a p^+ that may place mass on "impossible" events. Relaxation of (especially integer) constraints can result in low-complexity heuristics. An example of this is applying the decorrelator or MMSE filter for detection with a linear model with integer constraints.

A bank of *constrained detectors* ignores the interdependencies between the u_k. The detector for u_k updates the current prior marginal p_k based on the constraint $\mathcal{C}_k$ and $p(y \mid u_k)$. For convolutionally coded data, we may use the forward-backward algorithm. For a sequence of channel taps we may use a Kalman filter.

II. Convergence Analysis

We shall now consider an asynchronous K user CDMA system in the absence of multipath fading. Identical convolutional codes with free distance d_{free} are used by each transmitter.

We are interested in the effective noise variance at the output of each iteration. Considering an input noise variance v to the constrained data estimator (Viterbi decoder), we may bound the output noise variance v_d.

$$v_d \geq f(v) = 4 d_{\text{free}} Q\left(\sqrt{2 d_{\text{free}}/v}\right)$$

For a given spreading factor $\beta = K/N$, input variance v_d and thermal noise variance σ^2, the unconstrained joint detector described in [1] is characterized by $v_d = \beta v + \sigma^2$. This leads to the recurrence

$$v_d^{(m+1)} = F(v_d^{(m)}) \triangleq 4 d_{\text{free}} Q\left(\sqrt{\frac{2 d_{\text{free}}}{\beta v_d^{(m)} + \sigma^2}}\right),$$

In operating regions of interest, we may use the solutions to the fixed point equation $v_d = F(v_d)$ to accurately predict the performance. Furthermore $v_d^{(m)}$ may be used to predict the performance for finite number of iterations, m.

Given a fixed point solution x, we have the following sufficient condition for stability

$$0 < x < \frac{1}{\beta}\left(\frac{2 d_{\text{free}}}{3 \ln d_{\text{free}} - \ln \frac{4}{\pi}} - \sigma^2\right) \implies F'(x) < 1.$$

In practice, we have observed the existence of a stable fixed point close to the single user operating point and it is possible to derive an expression for the loss compared to single user for this point. Decoder failure occurs when a second fixed point appears at high noise variance. It can be shown that for high SNR this occurs for a critical value of β given by

$$\beta_{\text{crit}} = (2 - v)/f^{-1}(2).$$

We have verified this behavior with simulations.

References

[1] P. D. Alexander, A. J. Grant, and M. C. Reed, "Iterative detection on code-division multiple-access with error control coding," *European Transactions on Telecommunications*, vol. 9, no. 5, pp. 419–426, Sept.-Oct. 1998.