Design of Irregular Repeat-Accumulate Coded Physical-Layer Network Coding for Gaussian Two-way Relay Channels

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Abstract—This paper addresses the design of irregular repeat accumulate (IRA) codes for coded physical-layer network coding (PNC) for the binary-input Gaussian two-way relay channel, assuming perfect synchronization and equal received power at the relay. The design is based on a nontrivial extension of EXIT-chart based design. Specifically, we analyze the components of the IRA-PNC scheme and propose an approach to model the soft information exchanged between these components. Then, we develop upper and lower bounds on the extrinsic information transfer functions to characterize the iterative process of computing the network-coded information. Based on that, we construct optimized IRA codes to minimize the computation error at the relay. The optimized IRA-PNC has considerable performance improvement over the existing regular RA coded PNC. For a rate $\frac{3}{4}$ code, as an example, we observed improvements of 2.6 dB, and the optimized IRA-PNC scheme is only about 1.7 dB away from the capacity upper bound of the Gaussian two-way relay channel.

Index Terms—Two-way relay channel, physical-layer network coding, compute-and-forward, Repeat-Accumulate codes, convergence analysis, EXIT chart.

I. INTRODUCTION

Efficient communication over two-way relay channels (TWRCs) has attracted intensive research efforts, in light of the discovery of network coding [1]. From the literature, several classical relaying mechanisms have been extended to TWRCs. Among those, an amplify-and-forward based scheme, namely analog network coding [3–5], is simple to implement, but suffers from noise amplification and unnecessary power consumption [6]. A complete decoding-based scheme, where the relay fully decodes both users’ messages and then forms the network-coded message, can reduce the effect of noise amplification but suffers from multiplexing loss [2, 7, 8].

The physical-layer network coding (PNC) technique proposed in [6, 9, 10] can further improve the performance of a two-way relay system. Rather than decoding both users’ messages individually, the relay in a PNC scheme only computes and forwards [11] sufficient information to the two users, which can avoid the drawbacks of the analog network coding scheme and the complete decoding-based scheme. It has been shown in [7] from the information theoretic perspective that, the PNC scheme can achieve within $1/2$ bit of the capacity of the Gaussian TWRCs and is asymptotically optimal at a high signal-to-noise ratio (SNR).

The benefits of PNC have been recently demonstrated in various work. In [12], the authors showed that the PNC scheme achieves a higher maximum sum-rate, and a lower sum-bit error rate than the conventional transmission scheme for a number of practical scenarios. The work in [13] investigated the optimization of the PNC constellation with asynchronous transmission using QPSK and 16-QAM modulation, and demonstrated significantly improved end-to-end throughput. A modified higher order phase amplitude modulation was proposed in [14] to resolve the ambiguity mapping at the relay in a binary PNC scheme. Although most work assume perfect synchronization, [15] showed that even with synchronization errors, the PNC scheme can still provide a higher capacity than the complete decoding-based scheme. The work in [16–18] proposed several approaches to mitigate the effects brought by the asynchronous transmission in PNC schemes. A number of works also extended the PNC scheme to multiple-input multiple-output systems [19–25] or networks with multiple relays [26].

While most works considered uncoded PNC, [2] investigated a regular repeat-accumulated (RA) coded PNC scheme and proposed iterative message passing decoding algorithms for the channel coded PNC. Later, a convolutional coded PNC scheme with modified Viterbi and BCJR algorithms was studied in [27]. The work in [28] further extended the work in [13] to a convolutional-coded system. In [29], an asymptotically tight bound was derived for the error probability of a channel-coded PNC (CPNC) scheme, which showed that its error-rate performance is degraded by at most $\ln 2$ (in linear SNR scale) relative to the single-user case.

In this paper, we consider binary-input Gaussian TWRCs with perfect synchronization and equal power allocation for the two users, as in the pioneering work [2]. Applications of such a scenario can be found in satellite communications and deep-space communications [2, 6]. We investigate the PNC scheme from a binary channel code design’s perspective. The

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goal is to design a practical binary code for the CPNC scheme such that it performs close to the capacity limit of a binary-input Gaussian TWRC. In particular, we investigate irregular RA (IRA) coded PNC schemes for binary-input Gaussian TWRC. In the IRA-PNC scheme under consideration, the relay computes a binary network codeword, from its received noisy ternary superimposed signal sequence, which is then forwarded to the users. To carry out this computation, it is required to extend the conventional Tanner graph [30] to an equivalent Tanner graph (ETG), defined over a ternary superimposed signal domain [2]. The presence of the ternary signal leads to the challenges in the convergence analysis and design of a capacity-approaching IRA-PNC scheme.

In this paper, we address the challenges in the convergence analysis and design of IRA coded PNC schemes. Our main contributions are:

1. We analyze the component decoders of the IRA-PNC scheme and derive the generalized update rules for these components in terms of log-likelihood ratios (LLRs).

2. Then we propose two models for the soft information exchanged among the components decoders and develop bounds on the approximation of the extrinsic information transfer (EXIT) functions of the IRA-PNC scheme. Based on that, we carry out an EXIT chart curve-fitting technique to construct optimized IRA codes. Our developed IRA-PNC schemes have significantly improved performance compared to the existing regular RA coded PNC schemes. We show an example where the designed IRA-PNC scheme performs only 1.7 dB away from the capacity upper bound of the Gaussian TWRC, and outperforms the existing RA coded PNC scheme by 2.6 dB.

3. We compare our developed IRA-PNC scheme with a complete decoding-based network coding scheme, in which the relay completely decodes both users’ messages, using iterative multi-user detection and decoding, and then form the network-coded message. For fairness, both the CPNC scheme and the complete decoding-based scheme utilize their corresponding optimal codes and power allocations. Numerical results show that the CPNC scheme can significantly outperform the complete decoding-based scheme if the code rate is sufficiently high. This agrees with the information theoretic result in [7, 31].

The paper is organized as follows: Section II introduces the system model and the CPNC scheme. Section III analyzes the components of the IRA-PNC scheme and presents the computation operation. In Section IV, we perform an EXIT chart analysis and use curve-fitting techniques to optimize the degree distributions of the IRA-PNC scheme. In Section V, we present numerical results. Possible generalizations are presented in Section VI and Section VII concludes the paper.

II. SYSTEM MODEL

We consider a Gaussian TWRC where two single-antenna users, denoted by $A$ and $B$, exchange information via an intermediate single-antenna relay. The users and the relay operate in half-duplex mode and there is no direct link between the users. The transmission protocol under consideration employs two time-slots for each round of information exchange. In the first time-slot (uplink phase), the users transmit their signals to the relay. In the second time-slot (downlink phase), the relay broadcasts to the two users. At each node, the received signal is corrupted by additive white Gaussian noise (AWGN).

Now we illustrate the CPNC scheme for the binary-input Gaussian TWRC. The block diagrams of the CPNC scheme is depicted in Fig. 1, which follows [2]. We first consider the uplink phase. Let $b_A = [b_A(1), \ldots, b_A(k)] \in \{0, 1\}^k$ denote the length-$k$ binary message sequences of user $A$. This message sequence is encoded with a binary linear channel code and the resulting codeword is denoted by $c_A = [c_A(1), \ldots, c_A(n)] \in \{0, 1\}^n$, where $n$ denotes the length of the codeword. The code rate per user is given by $R = k/n$. The users’ codewords are modulated via binary phase shift keying (BPSK) ($0 \mapsto -1, 1 \mapsto +1$), resulting in the signal sequences $x_A = 2c_A - 1 \in \{-1, +1\}^n$. The encoder and modulation for user $B$ are the same as user $A$, with similar notations. The signal sequences of user $A$ and user $B$ are transmitted simultaneously.

It is noteworthy that, in general, the two users in a CPNC scheme of a TWRC may have different data rates and different signal power. In this paper, however, we will follow the pioneering work [2] by limiting our discussion to the cases where the two users have identical data rates and the same received symbol energies. Assuming perfect synchronization [2], the signal received by the relay is

$$y_R = \sqrt{E_s} x_A + \sqrt{E_s} x_B + n_R = \sqrt{E_s} (x_A + x_B) + n_R,$$

where $E_s$ is the received symbol energy per-user and $n_R$ is the AWGN vector at the relay. The variance of the noise is $\sigma^2$ and the per-user SNR in the uplink phase is given by $E_s/\sigma^2$.

In light of the notion of network coding [1], the relay needs to deliver a network-coded message to the two users. In this paper, we define the network-coded (NC) message sequence as $b_N = b_A \oplus b_B$, where “$\oplus$” denotes the element-wise modulo-2 addition operation. Upon receiving $y_R$, the task of the relay

![Fig. 1. Architecture of a two-way relay system operated with CPNC. The relay computes the network-coded message $b_N = b_A \oplus b_B$ without explicit decoding of both users’ individual messages. Here, “$+$” denotes the linear addition in real values and “$\oplus$” denotes the modulo-2 addition.](image)
is to compute an estimate of the NC message sequence $b_N$, given as
\[ \hat{b}_N = F_R(y_R). \]  

(2)

A computation error at the relay is declared if $\hat{b}_N \neq b_N$.

In the downlink phase, as shown in Fig. 1, the relay re-encodes the computed NC message sequence $\hat{b}_N$ into a codeword $c_R$, which is then BPSK-modulated, resulting signal sequence $x_R$. This signal is broadcast to the two users. Then, user $A$ first decodes the NC message $b_N$. If the NC message is correctly recovered by both the relay and user $A$, user $A$ can correctly recover user $B$’s message by performing
\[ \hat{b}_B = b_A \oplus \hat{b}_N, \]  

(3)

with the help of its own knowledge of $b_A$. In contrast, if a computation error happens at the relay, a decoding error will happen\(^1\). The operation at user $B$ is similar to that of user $A$. This completes one round of information exchange. More details about the downlink phase operation can be found in [2].

In the above CPNC scheme, the operation of decoding the NC message $b_N$ at each user in the downlink phase is a standard single-user decoding. Thus, the key issue in the decoding of the CPNC scheme is to efficiently compute the NC message $b_N$ at the relay in the uplink phase, i.e., Eq. (2). In this paper, we will only investigate the computation of the NC message $b_N$ at the relay (as in [2]).

### III. IRREGULAR REPEAT-ACCUMULATE CODED PHYSICAL-LAYER NETWORK CODING

In general, any binary linear channel code, such as a convolutional code [27], a turbo code and a low-density parity-check (LDPC) code [32], can be employed in the CPNC scheme. In this paper, we consider IRA codes, since their encoding is simpler than that of LDPC codes, and their structure allows a more flexible design than Turbo-codes. In particular, IRA codes have a flexible code structure, defined by the degree distribution of the variable nodes and check nodes, which allows for convenient design by curve-fitting in EXIT charts [33]. Here we consider non-systematic IRA codes. Our analysis and design also apply to a CPNC scheme with systematic IRA codes or other types of codes.

#### A. Encoding with an IRA Code

Consider the system in Section II. In the uplink phase, user $A$’s message bits $b_A(t)$, $t = 1, \ldots, k$, are repeated $d_v$ times, where $d_v \in \{2, 3, \ldots\}$ specifies the length of repetition. The repetition, or variable node (VN), degree distribution is given by $\lambda(d_v)$, $d_v \in \{2, 3, \ldots\}$ where $\lambda(d_v)$ is the portion of message bits with repetition length $d_v$. Notice that $\lambda(d_v) \geq 0$, $\sum_{d_v=2}^{\infty} \lambda(d_v) = 1$. The repeated bit sequence is permuted by a random interleaver, denoted by $\pi(\cdot)$. The interleaved sequence is encoded by a series of parity-check

\( ^1 \)There could be a minor case where the NC message $b_N$ is wrongly computed by the relay but the final decoding result at a user is correct. However, the probability of such a case vanishes as $k$ increases. We will not consider this trivial case in this paper.

#### B. Iterative Computation of the NC message $b_N$ at the Relay

The algorithm in [2] only applies to VNs of degree 3 and CNs of degree 2. Here, we develop a computation algorithm that applies to VNs and CNs of any degree. Then, we represent this algorithm in a log-likelihood ratio (LLR) format which will be required in the subsequent EXIT chart analysis.

Let us define $b_s = b_A + b_B \in \{0, 1, 2\}^k$ and $c_s = c_A + c_B \in \{0, 1, 2\}^n$ as a superimposed message sequence and a superimposed codeword, respectively. Consider the following linear processing

\[ y'_R = y_R + 2\sqrt{E_s} = 2\sqrt{E_s}c_A + c_B + n_R, \]  

(4)

where $y'_R$ is equivalent to $y_R$ for the purpose of computing $b_N$. From (4) we see that the signal $y'_R$ is a noisy copy of the superimposed codeword $c_s$. To compute the desired NC message, a “virtual encoding” process [2], which maps each superimposed message $b_s$ to a superimposed codeword $c_s$, is required. For an IRA coded PNC scheme, specifically, this “virtual encoding” process can be described via an equivalent Tanner graph (ETG), formed by superimposing two conventional Tanner graphs [2] of the same single-user IRA code, as shown in Fig. 2. The structure of the ETG resembles that of the single-user IRA code. However, there are two major differences:

1. The inputs and outputs have ternary symbols, i.e., 
   $b_s \in \{0, 1, 2\}^k$ and $c_s \in \{0, 1, 2\}^n$. The message codes of degrees $d_c$, where $d_c \in \{1, 2, \ldots\}$. The check node (CN) degree distribution is given by $\rho(d_c)$, $d_c \in \{1, 2, \ldots\}$ where $\rho(d_c)$ is the portion of CNs whose number of connected edges is $d_c + 1$. We denote the average CN and VN degrees by $\bar{d}_c$ and $\bar{d}_s$, respectively. The parity-check coded bits are then passed through an accumulator (ACC), generating the coded sequence $c_A$. The same operation is performed at user $B$. The irregularity of this code resides in various repetition lengths (VN degrees) and various CN degrees.

It is noteworthy that when the two users transmit with the same rate, the same code is employed [2]. Then, the modusum of the two users’ codewords is still a codeword of the code. This is to ensure that the relay is able to compute the linear network coded codeword without decoding individual user’s codeword, as we will see later.
exchanged between the component nodes consists of the probabilities for the ternary symbols.

2) The ETG features an equivalent CN function and an equivalent VN function, denoted by \(f_{\text{CN}}(\cdot)\) and \(f_{\text{VN}}(\cdot)\), respectively, which are different from those of the single-user code. As the receiver iterates, the computation process converges and a decision is made towards the estimated superimposed message sequence.

Given \(y_R\), the relay first exploits the ETG to compute an estimate of the ternary superimposed message sequence, denoted by \(\hat{b}_N\). Next, given \(\hat{b}_n\), the estimated NC message sequence \(B_N\) is obtained by calculating the modulo-2 of \(\hat{b}_n\), i.e.,

\[
\hat{b}_N(t) = \begin{cases} 
0 & \text{if } \hat{b}_n(t) = 0 \text{ or } 2, \\
1 & \text{if } \hat{b}_n(t) = 1,
\end{cases}
\]  

for \(t = 1, \cdots, k\).

Next we briefly illustrate how to iteratively compute the superimposed message sequence \(b_n\), based on the ETG given above. Consider a node in the ETG which has \(L\) edges. The ternary a priori message to the \(l\)th edge of this node, \(l = 1, \cdots, L\), is denoted by \(P_l(\cdot) = [p^1_0, p^1_1, p^1_2]\), in which \(p_{\theta}^l\) is the probability that the \(l\)th edge takes on the value of \(\theta\), \(\theta \in \{0, 1, 2\}\). The collection of \(P_l\) of all edges is denoted by \(P = \{P^{(1)}, \cdots, P^{(L)}\}\). In the iterative computation process, a node takes the a priori probabilities \(P\) to calculate the extrinsic probabilities, according to its update rule. For the \(l\)th edge, \(l = 1, \cdots, L\), the ternary extrinsic probabilities are denoted by \(Q_l = \{q^1_0, q^1_1, q^1_2\}\), and the collection of them for all edges is denoted by \(Q = \{Q^{(1)}, \cdots, Q^{(L)}\}\). The update rule can then be generally written as

\[
Q = f(P),
\]  

Here, we use “P” and “Q” to distinguish the a priori probabilities from the extrinsic probabilities.

Initially, the relay calculates the ternary intrinsic probabilities based on the channel observation \(y_R\):

\[
p^\text{CH}_0 = p(c_n = \theta|y_R) = \begin{cases} 
\gamma \exp \left(-\frac{(z_0 - \theta(2\sqrt{\text{SNR}))^2}{2}\right), & \theta = 0, 2, \\
2\gamma \exp \left(-\frac{(z_0 - \theta(2\sqrt{\text{SNR}))^2}{2}\right), & \theta = 1,
\end{cases}
\]  

where we have omitted the time-index, and \(\gamma\) is a normalization factor to ensure that \(p^\text{CH}_0 + p^\text{CH}_1 + p^\text{CH}_2 = 1\). These intrinsic probabilities are collected as \(P^\text{CH} = [p^\text{CH}_0, p^\text{CH}_1, p^\text{CH}_2]\), and they are only available to the accumulator. For the component decoders, the initialized a priori probabilities of each edge are \(P^{l(1)} = [1/4, 1/2, 1/4], l = 1, \cdots, L\) [2].

In the process of computing \(b_n\), the ternary messages are iteratively exchanged among the component nodes in the ETG, in a similar fashion as the conventional iterative decoding of the single-user IRA code. As the receiver iterates, the ternary messages are refined using the update rules (6) of the component nodes, which will be detailed next. After a number of iterations, the computation process converges and a decision is made towards the estimated superimposed message sequence \(\hat{b}_N\). Then the estimated NC message \(B_N\) can be obtained according to (5).

C. Update Rules

1) Update Rule with Probabilities: Let us first consider a CN with degree \(d_c = 2\). There are \(L = 3\) edges connected to this CN. Recall that the a priori messages available to the first and second edge are given by \(P^{(1)} = [p^1_0, p^1_1, p^1_2]\) and \(P^{(2)} = [p^2_0, p^2_1, p^2_2]\), respectively. The extrinsic message of the third edge, denoted by \(Q^{(3)} = [q^3_0, q^3_1, q^3_2]\), can be obtained as \(Q^{(3)} = f_{\text{CN}}^2(P^{(1)}, P^{(2)})\), where the update rule \(f_{\text{CN}}^2(\cdot)\) for a CN of degree 2 is given by [2]

\[
q^3_0 = \gamma (p^1_0 p^2_0 + p^1_1 p^2_1 + p^1_2 p^2_2),
\]  

\[
q^3_1 = \gamma (p^1_1 p^2_0 + p^1_2 p^2_1 + p^1_0 p^2_2),
\]  

\[
q^3_2 = \gamma (p^1_2 p^2_0 + p^1_0 p^2_1 + p^1_1 p^2_2).
\]

Here, \(\gamma\) is a normalization factor to ensure that \(q^3_0 + q^3_1 + q^3_2 = 1\).

In general, for a CN with a degree \(d_c > 2\), the update function \(f_{\text{CN}}^d(\cdot)\) can be obtained by successively utilizing the degree-2 CN update rule, given by

\[
\Gamma^{(2)} = f_{\text{CN}}^2(P^{(1)}, P^{(2)}),
\]

\[
\Gamma^{(l)} = f_{\text{CN}}^2(\Gamma^{(l-1)}, P^{(l)}),
\]

\[
Q^{d_c+1} = \Gamma^{(d_c)} = f_{\text{CN}}^2(\Gamma^{(d_c-1)}, P^{(d_c)}).
\]

We refer to the above approach as a successive update.

2) Update Rule with LLRs: The ternary probabilities exchanged in the CPNC decoders put challenges on the analysis and design of the scheme. We next represent the update rule in terms of LLRs, which will be required in our subsequent EXIT chart analysis. For the \(l\)th edge of a component node in the ETG, the LLR couple associated with the a priori (ternary) probabilities are defined as

\[
\Lambda_p^{(l)} \triangleq \log \left(\frac{p^0_0 + p^0_2}{p^1_0} \right) \text{ and } \Omega_p^{(l)} \triangleq \log \left(\frac{p^0_0}{p^2_0} \right),
\]  

which are sufficient statistics of \(p_0, p_1, p_2\).

From (5), we see that values \(b_n(t) = 0\) and \(b_n(t) = 2\) of the superimposed message are both mapped to the NC message bit \(b_N(t) = 0\). Therefore, \(\Lambda_p^{(l)}\) is related to the LLR of the binary NC message bit, and it has a pivotal role in the iterative computation process. To distinguish the two LLR values in (14), we refer to \(\Lambda_p^{(l)}\) as the primary LLR and \(\Omega_p^{(l)}\) as the

Following the common approach in literature for IRA codes [33], a CN with degree \(d_c\) has \(d_c\) edges connected to the interleaver and one additional edge connected to the ACC. A VN with degree \(d_v\) has \(d_v\) edges connected to the interleaver.
\[
\Lambda_Q^{(3)} = \log \left( \frac{q_0^{(3)} + q_2^{(3)}}{q_1^{(3)}} \right) = \log \left( \frac{\left( p_0^{(1)} p_0^{(2)} + \frac{p_1^{(1)} p_1^{(2)}}{2} \right) + \frac{p_1^{(1)} p_1^{(2)}}{2} + \frac{p_0^{(1)} p_0^{(2)}}{2}}{p_2^{(1)} p_2^{(2)} + \frac{p_1^{(1)} p_1^{(2)}}{2} + \frac{p_0^{(1)} p_0^{(2)}}{2}} \right),
\]
(11)

\[
\left[ \Lambda_Q^{(3)}, \Omega_Q^{(3)} \right] = f_{CN}^2 \left( \left[ \Lambda_p^{(1)}, \Omega_p^{(1)} \right], \left[ \Lambda_p^{(2)}, \Omega_p^{(2)} \right] \right),
\]
(12)

\[
\left[ \Lambda_Q^{(l)}, \Omega_Q^{(l)} \right] = f_{VN}^d \left( \left[ \Lambda_p^{(1)}, \Omega_p^{(1)} \right], \ldots, \left[ \Lambda_p^{(l-1)}, \Omega_p^{(l-1)} \right], \left[ \Lambda_p^{(l+1)}, \Omega_p^{(l+1)} \right], \ldots, \left[ \Lambda_p^{(d_v+1)}, \Omega_p^{(d_v+1)} \right] \right),
\]
(13)

** secondary LLR. Similarly, the primary and secondary LLRs associated with the extrinsic probabilities are defined as

\[
\Lambda_Q^{(l)} = \log \left( \frac{q_0^{(l)} + q_2^{(l)}}{q_1^{(l)}} \right) \quad \text{and} \quad \Omega_Q^{(l)} = \log \left( \frac{q_0^{(l)}}{q_2^{(l)}} \right).
\]
(15)

Consider a CN with \( d_c = 2 \). The primary extrinsic LLR of the third edge is calculated by (11) where \( \equiv \) follows from (8)-(10). The secondary extrinsic LLR is calculated as

\[
\Omega_Q^{(3)} = \log \left( \frac{q_0^{(3)}}{q_2^{(3)}} \right),
\]
(16)

where

\[
K_{CN} = \frac{1 + \exp \left( \Omega_p^{(1)} \right) \left[ 1 + \exp \left( \Omega_p^{(2)} \right) \right]}{2 \exp \left( \Lambda_p^{(1)} \right) \exp \left( \Lambda_p^{(2)} \right)}.
\]
(17)

Now, the update rule in terms of LLRs of a CN of \( d_c = 2 \) is given by (12). The update rule of a CN with \( d_c > 2 \) can be calculated using the successive update approach described previously. In the sequel, we will use \( f_{CN}^d (\cdot) \) to denote the update rule of a CN of degree \( d_c \) in LLRs. A property of the update rule of a CN is presented next, which will be used later in the next section.

**Property 1:** For a CN with degree \( d_c \), we have the output secondary LLR \( \Omega_Q^{(l)} = 0 \) as long as there exists an edge \( l' \), \( l' \neq l \), such that the input secondary LLR \( \Omega_{Q}^{(l')} = 0 \).

**Explanation:** In (16), if any of \( \Omega_p^{(1)} \) or \( \Omega_p^{(2)} \) equals to zero, the term \( \Omega_Q^{(3)} \) will be zero. Consider the successive update rule, we obtain Property 1.

The derivation of the VN update rule \( f_{VN} (\cdot) \) in LLRs is similar and it is given by (13) where

\[
K_{VN} = \log \left( \frac{1 + \prod_{l=1,l \neq l'}^{d_v} \exp \left( \Omega_p^{(l')} \right)}{\prod_{l=1,l \neq l'}^{d_v} \left( 1 + \exp \left( \Omega_p^{(l')} \right) \right)} \right).
\]
(18)

The derivation is given in Appendix A.

**IV. CONVERGENCE BEHAVIOR ANALYSIS AND OPTIMIZATION OF IRA-PNC SCHEMES**

It is well-known that in the conventional single-user AWGN channel, the performance of an iteratively decoded IRA code is largely affected by its VN degree distribution \( \lambda(d_c) \), and the CN degree distribution \( \rho(d_c) \). The optimal performance can be achieved using the EXIT chart curve-fitting technique [33]. Now, we adopt this methodology in designing the IRA-PNC scheme, so as to approach the capacity limit of the binary-input Gaussian TWRC. However, for the two-user CPNC scheme, there lacks a method to characterize the exiting behaviors w.r.t. the ternary probabilities, that are exchanged in the iterative computation process.

In this section, we first propose a method to model the soft information exchanged among the components of the IRA-PNC scheme. This will enable us to obtain upper and lower bounds on the approximation of the EXIT functions. Based on that, we design optimal component codes via curve-fitting.

**A. Modeling of EXIT Functions**

To carry out convergence behavior analysis, we partition the ETG of the IRA-PNC scheme into two parts: an inner component decoder consisting of the combined CN and ACC (CN-ACC) decoder, and an outer component decoder consisting of the VN decoder. The idea of the EXIT chart technique is to
predict the behavior of the iterative process by solely looking at the input/output mutual information of the two individual component decoders of the CPNC scheme.

Unlike the decoding of a conventional binary IRA code where binary probabilities are exchanged between the component decoders, the soft information exchanged between the CPNC component decoders has a ternary form. The ternary probabilities (or soft information) of the CPNC scheme can also be written in terms of the primary LLR $\Lambda$ and the secondary LLR $\Omega$, as in the previous section. In particular, the primary LLR $\Lambda$ is related to the NC message $b_N$ to be computed. For simplicity, we omit the time index here. We denote $b_N$ the random variable w.r.t. the NC message bit and denote $\Lambda$ the random variable w.r.t. the primary LLR. Thus, the mutual information between $b_N$ and the input primary LLR $\Lambda_P$, $I_E = I(b_N; \Lambda_P)$, will be used for tracking the a priori information of a component decoder of the CPNC scheme. Similarly, the mutual information between $b_N$ and the output primary extrinsic LLR $\Lambda_Q$, $I_E = I(b_N; \Lambda_Q)$, will be used for tracking the corresponding extrinsic information. An output mutual information of $I_E = 1$ means that all NC message bits $b_N$ can be decoded error free.

The relationship of the input-output mutual information, i.e., the EXIT function, of the inner component decoder (CN-ACC decoder) with CN degree distribution $\rho(d_c)$ can be written as

$$I_E = T_{\text{Inner}}(I_A, \mathbb{P}(\Omega_P), \rho(d_c), E_s/\sigma^2),$$

where $\mathbb{P}(\Omega_P)$ denotes the probability density function (PDF) of the secondary LLR $\Omega_P$. We remark that, unlike the conventional single-user case, the EXIT function of the CPNC scheme is also affected by the PDF of the secondary LLR $\Omega_P$. Similarly, the EXIT function of the outer component decoder (VN decoder) with VN degree distribution $\lambda(d_v)$ can be written as

$$I_E = T_{\text{Outer}}(I_A, \mathbb{P}(\Omega_P), \lambda(d_v)).$$

Notice that the EXIT function of the VN decoder is not affected by the SNR, since it is not directly connected to the channel observation.

Numerical results show that the PDF of the primary LLR approaches a consistent Gaussian-like distribution [34] with its mean equal to half of its variance, with the increasing number of iterations, as shown in Fig. 3. Thus, similar to [34], we can approximate the primary a priori LLR as

$$\Lambda_P = \frac{\sigma_A^2}{2}(1 - 2b_N) + n_A,$$

where $n_A$ is a Gaussian random variable with variance $\sigma_A^2$. However, the PDF of the secondary LLR, as shown in Fig. 3, is not a Gaussian-like distribution. This makes the analytical treatment of the EXIT functions difficult. In order to tackle this problem, we propose two models for the secondary a priori information.

**Model I**: We assume that perfect secondary LLR is available in this model, that is,

$$\tilde{\Omega}_P = \begin{cases} +\Psi & \text{if } b_s = 0, \\ 0 & \text{if } b_s = 1, \\ -\Psi & \text{if } b_s = 2, \end{cases}$$

where $\Psi$ denotes a large positive value, e.g., 30, used in our simulation. Since the actual decoding process does not have perfect a priori information on the secondary LLR $\Omega_P$ for the component decoders, we have $I_E = T_{\text{Inner}}(I_A, \mathbb{P}(\Omega_P), \rho(d_c), E_s/\sigma^2) \leq T_{\text{Inner}}(I_A, \mathbb{P}(\tilde{\Omega}_P), \rho(d_c), E_s/\sigma^2)$ for the inner component decoder (CN-ACC decoder). Thus, we can obtain an upper bound for the approximation of the EXIT function of the inner component decoder by using Model I. Similarly, we can obtain an upper bound on the approximation of the EXIT function of the outer component decoder using Model I.

**Model II**: We assume the a priori secondary LLR $\Omega_P$ is completely absent, i.e., $\tilde{\Omega}_P = 0$.

As the actual decoding retains certain a priori information on the secondary LLR $\Omega_P$ for the component decoders, setting $\Omega_P$ to zero will result in an information loss. Following the data processing inequality [35], we have $I_E = T_{\text{Inner}}(I_A, \mathbb{P}(\Omega_P), \rho(d_c), E_s/\sigma^2) \geq T_{\text{Inner}}(I_A, \mathbb{P}(\tilde{\Omega}_P), \rho(d_c), E_s/\sigma^2)$ for the inner component decoder and this also applies to the outer component decoder. Thus, a lower bound of the approximation of the EXIT function can be obtained using Model II.

### B. EXIT Charts

We next show the EXIT functions of the component decoders of the IRA-PNC scheme using the two a priori information models developed earlier. The EXIT functions of the inner CN-ACC decoder with CN degrees $d_c = 1, \ldots, 5$ are shown in Fig. 4. These EXIT functions are obtained via simulations where Model I and Model II are used to construct the a priori information. Clearly, the EXIT function obtained with Model I is always higher than with Model II. This suggests that the availability of the secondary LLR $\Omega_P$ can contribute to a higher output extrinsic information. From Fig. 4, we also observe that the gap between the EXIT functions with Model

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3Here we have omitted the time-index for simplicity.
I and with Model II diminishes as the CN degree $d_c$ increases. When $d_c \geq 2$, the gap is almost unnoticeable. Here we give an intuitive explanation for this behavior. Let us consider the \textit{a priori} information model I, the probability of $\Omega_F = 0$ is 0.5 since 50\% of message bits have $b_Y = 1$. Recall Property 1 which states that the output secondary LLR of a CN is zero as long as one of its input edges has the secondary LLR equal to zero. As the CN degree $d_c$ increases, the probability of this event (there exists one edge whose secondary LLR is zero) also increases. As a result, there will be more zero-secondary LLR at the output of the CN nodes. This will restrain the propagation of the secondary LLR from the CN to the ACC of the inner decoder. As the CN degree becomes very high, the propagation of the secondary LLR becomes minimal and Model I and Model II tends to be identical.

From [33], it is known that to minimize the area between the EXIT functions of the component decoders, a capacity achieving IRA code tends to have a fairly large average CN degree, i.e., $\bar{d_c} > 2$. In this circumstance, for inner component decoder, the upper bound (Model I) and lower bound (Model II) of the approximation of the EXIT functions overlaps with each other. Therefore, either Model I or II can be used to obtain the approximation of the EXIT function of the inner component. Similarly, we can also use Model I or Model II to obtain the upper and lower bounds on the approximation of the EXIT function of the outer component decoder. We remark that since Model II gives a lower bound on the approximation of the EXIT function for either the inner or the outer component decoder, an optimal code based on Model II can always have its convergence guaranteed when the SNR is above its designed threshold.

\textbf{Example 1:} We consider an IRA-PNC scheme with per-user code rate of $R = 1/3$. In particular, the IRA under consideration has an average CN degree $\bar{d_c} = 2.4$ and an average VN degree $\bar{d_v} = 7.2$. The code parameters are given in Table I. In Fig. 5, we plot the EXIT function obtained by using the \textit{a priori} information Model II and the actual decoding trajectory obtained from simulation. We observe that using the \textit{a priori} information Model II, the EXIT functions of the component decoders of the IRA-PNC scheme can be accurately characterized. In the sequel, we will focus on using \textit{a priori} information Model II for the design of the IRA-PNC scheme.

\textbf{C. Code Optimization via Curve-fitting of the EXIT functions}

Based on the developed EXIT functions of the component decoders of the IRA-PNC scheme, we now adopt the EXIT chart curve-fitting technique to design optimal IRA codes. The goal is to find CN and VN degree distributions such that the gap between the EXIT curves of the inner component decoder and the outer component decoder is minimized. Similar to [33], we first select an appropriate CN degree distribution. Then, we fit the EXIT curve of VN decoder to that of the CN-ACC decoder, by optimizing the degree distribution of the VN decoder via linear programming.

We next show two examples of the code design via EXIT curve-fitting for the IRA-PNC scheme. To avoid redundancy, the implementation details of the curve-fitting are omitted and can be found in [33]. In this paper, we restrict ourselves to the commonly used concentrated check degree distributions [36, Section 3.17].

\textbf{Example 2:} We consider an IRA-PNC scheme where the code rate of each user is $R = 3/4$. In a conventional single-user case, it is known that, for a non-systematic RA code, a non-zero fraction of the CNs should have degree one to ensure that its decoder makes progress in the first iteration [33, 37]. From (12), we notice that in the IRA-PNC scheme, the CN degree distribution should contain a non-zero fraction for degree one CNs, similar to the conventional single-user RA codes case. In this design example, the choice of the portion of $d_c = 1$ CNs follows from the convention in [33]. The details are provided in Table I. In addition, to carry out EXIT curve-fitting, flexibility of the VN nodes are required so that the average VN degree cannot be too small, e.g., $\bar{d_v} > 3$. Then, for a code rate of $3/4$, the average CN degree should be large enough, e.g., $\bar{d_c} > 2$. In this setting, the EXIT function of the bi-regular code can be characterized by Model II.
The performance improvement is slight in this case. We next explain why the CPNC scheme with the designed IRA code only slightly outperforms that with a regular code. We also give in Table I. We see that for the case of R = 3/4, the threshold is found to be at E_b/N_0 = 2.1 dB. This shows that our developed IRA-PNC scheme can significantly outperform the bi-regular RA coded PNC scheme. The performance improvement is obtained from fitting the EXIT functions, and we refer to this performance improvement as a curve-fitting gain. In this example, the curve-fitting gain is about 2.6 dB.

Example 3: We consider another case where R = 1/3. For a regular RA coded PNC scheme, the threshold is found to be at E_b/N_0 = 2.2 dB, as shown in Fig. 7. We construct an IRA code for the CPNC scheme, using the curve-fitting technique based on our developed EXIT functions. The decoding threshold is found to be at E_b/N_0 = 2.1 dB. The degree distributions of our designed IRA code for the CPNC scheme are also given in Table I. We see that for the case of R = 1/3, the CPNC scheme with the designed IRA code only slightly outperforms that with a regular code. We next explain why the performance improvement is slight in this case.

Consider a simplified computation approach in which the secondary LLR Ω_p is always set to zero in the iterative computation process. Then, from (14), the soft information exchanged in the CPNC components can be completely specified by \([p_0 + p_2, p_1]\), which has two elements. Here, we refer to this simplified approach as iterative computation with binary information exchange. In contrast, we refer to the approach utilizing both the primary and secondary LLRs as iterative computation with ternary information exchange, since the exchanged soft information has three elements, see (14).

The performance improvement of using ternary information exchange (which utilizes the secondary LLR), over that with binary information exchange (which does not use the secondary LLR), is referred to as the secondary LLR gain. For the CPNC scheme of per-user coding rate R = 1/3, it is shown that the secondary LLR gain is as much as 0.5 dB when a regular RA code is utilized (see Fig. 8). In the process of optimizing the degree distributions of the IRA code to obtain the curve-fitting gain, the inner component decoder tends to have a relatively large average CN degree. This results in a reduced secondary LLR gain, as discussed in Section IV.B. For a relatively large CN degree, the secondary LLR gain vanishes. Finally, the combined effect of increased curve-fitting gain and reduced secondary LLR gain leads to only a slight performance improvement.

In contrast, in Example 2 where R = 3/4, the average degree of the CN of the CPNC scheme with a regular/bi-regular RA code is already relatively large, e.g., \(\bar{d}_c \geq 2\). In this case, the secondary gain is already fairly small, as we can see in Fig. 4. Therefore, as we carry out the curve-fitting, there is no loss in the secondary LLR gain and the curve-fitting gain leads to a significant performance improvement.

It is noteworthy that IRA codes are special LDPC codes with a simpler encoder than general LDPC codes but with similar performance. The code optimization technique proposed in this paper may apply to general LDPC codes.

V. SIMULATION RESULTS

In the previous section, we have designed IRA-PNC schemes based on EXIT chart analysis and curve-fitting techniques. In this section, we present numerical results to show the benefits of our designed IRA-PNC schemes for finite code lengths. Specifically, we first compare the bit-error rate (BER) performance of our developed IRA-PNC schemes to the existing CPNC schemes with regular (or bi-regular) RA codes. Next, we compare the performance of our developed IRA-PNC scheme to the capacity limits, as well as to the complete decoding-based scheme.

In the simulations, we consider the BER performance of computing the NC message b_k at the relay. In all simulations, the length of the binary message sequence of each user is set to k = 32768. In the iterative computation process, the maximum number of iterations is set to 200.
A. IRA-PNC Schemes versus Regular (or Bi-regular) coded PNC Schemes

1) Per-user Code Rate \( R = 3/4 \): The BER simulation results of the CPNC scheme with this code rate are shown in Fig. 8. At a BER of \( 10^{-4} \), our developed IRA coded PNC scheme performs about 2.6 dB better than the bi-regular RA coded PNC scheme. This is in line with our EXIT chart analysis. From this result, we can conclude that IRA codes designed based on our EXIT analysis can significantly improve the performance of the CPNC scheme. We also notice that there is no performance degradation when the iterative computation with ternary information exchange is replaced by that with the binary information exchange. This behavior has been explained in Section IV.C.

2) Per-user Code Rate \( R = 1/3 \): The BER simulation results of the CPNC scheme with this code rate are shown in Fig. 8. When the iterative computation with binary information exchange is utilized, our IRA-PNC scheme is about 0.5 dB better than the existing PNC scheme with the regular code in [2]. The performance improvement is from the full realization of the curve-fitting gain. When ternary information exchange is utilized, the designed IRA-PNC scheme is about 0.1 dB better than that with the regular RA code. These results are also in line with our EXIT chart analysis.

3) Other Code Rates: Fig. 9 shows the performance of the optimized IRA-PNC scheme with various code rates, where ternary information exchange is utilized. For code rates of 1/2 and 2/3, at a BER of \( 10^{-4} \), we observe that the performance improvement over regular RA-PNC schemes are 1.6 dB and 1.9 dB, respectively. The code parameters are given in Table I.

B. CPNC Schemes Versus Complete Decoding-Based Network Coding Schemes

Now, we compare the performance of the CPNC scheme to the scheme which performs complete decoding to generate the NC message at the relay. For a fair comparison, the optimized IRA-PNC scheme and the optimized complete decoding-based scheme are considered. In particular, given a total power constraint, equal power allocation is the best for a PNC scheme, see [2, 38]. On the other hand, unequal power allocation is optimal for the complete decoding-based scheme [41, 42], as this facilitates the complete separation of two users’ codewords.

For the scheme with complete decoding, we employ iterative multi-user detection and decoding (IDD) [41] to fully decode both user \( A \) and user \( B \)'s messages \( b_A \) and \( b_B \), and then determine the NC message as \( b_A \oplus b_B \). In an IDD algorithm, soft information is iteratively exchanged between multiple single user decoders and a multi-user detector; details for code optimization using IDD algorithm can be found in [42, 43]. We emphasize that the “CNC1” scheme in [2] is equivalent to the complete decoding scheme (considered in this paper) with no iteration between the multi-user detector and the decoder. The performance loss of not using the IDD, however, can be up to several dB in power efficiency. For a fair comparison, in the scheme with complete decoding at the relay, we use the optimal power allocation between the two users [35, 42] and optimize its IRA code for degree distributions. The optimized code is provided in Table I.

In Fig. 10, we compare the performance of our IRA-PNC scheme and the complete decoding-based scheme, with \( R = 3/4 \). The optimized power allocation ratio for the complete decoding-based scheme is 3.2 at this code rate. The capacity limit for the complete decoding-based scheme is found to be at \( E_b/N_0 = 4.3 \) dB. The limit from the cut-set capacity upper bound of [7] of a Gaussian TWRC with binary inputs is found to be at \( E_b/N_0 = 1.67 \) dB. Note that the capacity limits for both schemes are for binary inputs with BPSK modulation. At BER = \( 10^{-4} \), our developed IRA-PNC scheme is about 1.7 dB away from the capacity upper bound. At BER = \( 10^{-4} \), the IRA-PNC scheme is about 2 dB better than the optimized complete decoding-based scheme. Note that, for this case, our designed IRA-PNC scheme clearly outperforms the capacity limitation.
Fig. 10. Comparison between the CPNC scheme and the complete decoding-based scheme where $R = 3/4$.

Fig. 11. Comparison between CPNC scheme the complete decoding-based scheme where $R = 1/3$.

limit of the complete decoding-based scheme.

In Fig. 11, we compare the performance of the IRA-PNC scheme and the complete decoding-based scheme where $R = 1/3$. The optimized power allocation ratio for the complete decoding-based scheme with IDD is about 0.6 dB better than the optimized IRA-PNC scheme. This shows that PNC with compute-and-forward may not be a good choice when the code rate is low. This is in line with the information theoretic result [7, 31], which shows that complete decoding-based scheme can outperform the CPNC scheme in terms of their achievable rates, as the SNR or coding rate becomes small. In Fig. 11, we also include the performance of the CNC1 scheme discussed in [2], which is equivalent to the complete decoding-based scheme but without iteration between the detector and decoders. The CNC1 suffers from a loss of about 1 dB relative to the complete decoding-based scheme with IDD. Due to this loss, the CNC1 scheme performs worse than the IRA-PNC scheme. We emphasize that when the complete decoding-based scheme is properly designed, it outperforms the CPNC scheme at $R = 1/3$.

VI. DISCUSSION OF GENERALIZATIONS

Here, we briefly discuss how to extend our CPNC design method to cases without equal receive power and perfect synchronization. For the case with unequal receive power for the two users, the relay observes a quaternary signal, instead of the ternary one as in the equal power case. In this case, we may still use the primary LLR to characterize the convergence behavior of the network coded bits. As opposed to the equal power case, however, two secondary LLRs have to be analyzed regarding their impact on the convergence behavior of the primary LLR. The optimization of the code parameters will be similar. For BPSK modulation, imperfect synchronization will have further effects on the two users’ received power, which can be treated with additional secondary LLRs. In this case, a similar approach to the unequal received power case may be applied. For QPSK modulation, the approaches developed in [13, 16, 38] may be applied to deal with the asynchronous phase problem. For example, in [16], an improved belief propagation algorithm, based on over-sampling of the continuous signal, was introduced to deal with asynchronous carrier phase. This technique may be incorporated in our design of the CPNC scheme in the case with carrier phase asynchrony.

The work in this paper may be extended to higher-order modulation schemes, like 4-PAM and 16-QAM. This can be done by employing non-binary RA coded high-level modulations. In this case, the code optimization approach proposed in this paper needs to be modified for the non-binary codes. For example, for the non-binary RA coded high-level modulation, we need to define and track multiple primary LLRs and multiple secondary LLRs, and evaluate the contribution of the secondary LLRs to the primary LLRs in the iterative decoding process.

In this paper, we only focused on the symmetric case where the two users have the same code rate. For an asymmetric case, e.g., where user A has a higher code rate than user B, we may consider that user B utilizes a subset of user A’s codewords. This will result in a nested code [39, 40]. Then the proposed code optimization technique can be applied to find the optimum code parameters. These extensions are beyond the scope of this paper and will be considered in future works.

VII. CONCLUSION

In this paper, we developed an IRA coded PNC scheme for binary input Gaussian TWRCs. We extended the EXIT chart technique to analyze the convergence behavior of the iterative computation process of the IRA-PNC scheme. Based on that, we optimized the degree distributions of the components of the IRA-PNC scheme. Our optimized IRA-PNC scheme significantly outperforms existing regular (or bi-regular) RA coded PNC schemes. We also showed that a CPNC scheme has the most significant benefit when the code rate is high. Interestingly, in a high coding rate regime, the performance improvement of using our EXIT curve-fitting to design an IRA coded PNC scheme is most significant. We also noted that, for a very low SNR or code rate, CPNC scheme is worse than the complete decoding based scheme with iterative multi-user
\[
\Lambda_Q^{(1)} = \log \left( \frac{q_0^{(1)} + q_2^{(1)}}{q_1^{(1)}} \right) = \log \left( \frac{4^{(d_v-2)} \cdot \prod_{l=2}^{d_v} p_0^{(l)} + 4^{(d_v-2)} \cdot \prod_{l=2}^{d_v} p_2^{(l)}}{2^{(d_v-2)} \cdot \prod_{l=2}^{d_v} p_1^{(l)}} \right)
\]

\[
= (d_v - 2) \log 2 + \sum_{l=2}^{d_v} \Lambda_P^{(l)} + K_{VN}.
\]

appendix A

DERIVATION OF VN UPDATE RULE

Consider a VN with degree \(d_v\). Without loss of generality, we consider the update of the extrinsic information on the first edge, based on the a priori information from the edges with index 2 to \(d_v\).

Borrowing the result Eq. (13) of [2] and extending it to \(d_v - 1\) edges, we have

\[
q_0^{(1)} = \gamma \cdot 4^{(d_v-2)} \cdot \prod_{l=2}^{d_v} p_0^{(l)},
\]

\[
q_1^{(1)} = \gamma \cdot 2^{(d_v-2)} \cdot \prod_{l=2}^{d_v} p_1^{(l)},
\]

\[
q_2^{(1)} = \gamma \cdot 4^{(d_v-2)} \cdot \prod_{l=2}^{d_v} p_2^{(l)},
\]

where \(\gamma\) is for normalization purpose. The primary LLR \(\Lambda_Q^{(1)}\) is calculated by (23) where

\[
K_{VN} = \log \left( \frac{1 + \prod_{l=2}^{d_v} \exp \left( \Omega_P^{(l)} \right)}{\prod_{l=2}^{d_v} \left( 1 + \exp \left( \Omega_P^{(l)} \right) \right)} \right).
\]

The derivation of the secondary LLR \(\Omega_Q^{(1)}\) can be carried similarly.

REFERENCES


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TABLE I
Code Parameters

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Ingmar Land is Senior Research Fellow at the Institute for Telecommunications Research (ITR), University of South Australia. Before joining ITR in 2007, he was Assistant Professor for Communication Theory at Aalborg University, Denmark. He received his Dr.-Ing in 2004 from the University of Kiel, Germany, and he studied for his Dipl.-Ing. at the University of Ulm, Germany, and at the University of Erlangen-Nürnberg, Germany. His research interests are coding and information theory with application to cooperative communications, multiuser communication, distributive source coding and physical-layer security.