Concatenated Space-Time Coding

Christian Schlegel
Dept. of Electrical Engineering
University of Utah
Salt Lake City, UT 84112
schlegel@ee.utah.edu

Alex Grant
Institute for Telecommunications Research
University of South Australia
Mawson Lakes, SA 5095
alex.grant@unisa.edu.au

Abstract— **The serial concatenation of standard convolutional codes with differential space-time modulation is considered for fast flat fading multiple antenna channels. Decoding is performed iteratively by passing symbol-wise *a-posteriori* probability information between the decoders of the inner space-time code and the outer convolutional code. An input-output extrinsic information transfer analysis is used to predict thresholds for outer codes of various memory orders. Simulation results show that this system can achieve bit error rates below 10^{-4} at less than 2.5 dB from the Shannon capacity of the multiple antenna channel.**

I. Introduction

Spatial diversity has been proposed for support of very high rate data users within third generation wide-band Code-Division Multiple Access (CDMA) systems such as cdma2000 [1]. Using multiple antennas, these systems achieve gains in link quality and therefore capacity. Classically, diversity has been exploited through the use of either beam-steering, or through diversity combining [2, 3].

More recently it has been realized that co-ordinated use of diversity can be achieved through the use of space-time codes. Such systems can theoretically increase capacity by up to a factor equaling the number of transmit and receive antennas in the array [4–8]. Rather than relying solely on array processing of uncoded transmissions, forward error correction codes which add redundancy in both the temporal and spatial domains are designed specifically for channels with multiple transmit and receive antennas.

There are currently two main approaches to realizing the capacity potential of these channels: *coordinated* space-time codes and *layered* space-time codes.

Coordinated space-time block codes [9–11] and trellis codes [12–16] are are designed for coordinated use in space, and time. The data is encoded using multi-dimensional codes that span the transmit array. Trellis codes are typically decoded using the Viterbi algorithm. A serious obstacle to extension to larger arrays however is the rapid growth of decoder complexity with array size and data rate: the number of states in a full-diversity space-time trellis code for t transmit antennas with rate R is $2^{(t-1)R}$. The other approach uses layered space-time codes [17–19], where the channel is decomposed into parallel single-input, single-output channels. The receiver successively decodes these layers by using antenna array techniques and linear or non-linear cancellation methods. Both approaches show much promise, but the latter is more scalable. It also has the advantage that available technology such as standard error control codecs can be more easily integrated.

Supported in part by NSF Grants CCR 9732962 and ECS 9979323.

Much of the space-time coding literature assumes the availability of good channel estimates, which are required for decoding. In the absence of channel knowledge, the capacity gains to be achieved depend upon the coherence time of the channel [20–22]. More recently, there has been considerable effort to design space-time codes that operate in the absence of channel state information [23–33]. Typically, these codes are coordinated space-time block codes whose symbols are unitary matrices. Such codes may also be used as part of differential space-time modulation schemes.

In this paper, we pursue an approach motivated by turbo decoding methods. The main idea is to use a serially concatenated coding system in which the “inner code” is a short space-time block code that spans both space and time. The “outer code” consists of a standard high-rate convolutional code. We use extrinsic information transfer charts to determine operation thresholds for these concatenated codes, which are compared with simulation. Operation thresholds about 2 dB from channel capacity are obtained.

II. System Model

Transmission takes place over a channel with t transmit antennas and r receive antennas. At time $k = 1, \ldots, n$, each transmit antenna $i = 1, \ldots, t$ selects a complex symbol c_{ik}, which is modulated onto a pulse waveform and transmitted over the channel. Taken as a vector, $c_{1k}, c_{2k}, \ldots, c_{tk}$ is referred to as a *space-time symbol*. At each receive antenna $j = 1, \ldots, r$, the signal is passed through a filter matched to the pulse waveform and sampled synchronously. If the channel delay spread is negligible and fading conditions are approximately constant over n symbols, the samples taken by receive antenna j can be modeled as $y_{jk} = \sqrt{\rho/t}\sum_{i=1}^{t} H_{ji}c_{ik} + n_{jk}$, where H_{ji} is the complex fading path gain from transmit antenna i to receive antenna j, n_{jk} is a complex circularly symmetric Gaussian noise sample, and ρ is the signal-to-noise ratio per receive antenna.

It is customary to collect the transmitted space-time symbols into a $t \times n$ codeword matrix, $\mathbf{C} = \{c_{ik}\}$, where rows correspond to different transmit antennas and columns correspond to different times. Considering a sequence of L codeword transmissions $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_L$, the channel can be written as

$$\mathbf{Y}_l = \sqrt{\frac{\rho}{t}}\mathbf{H}_l\mathbf{C}_l + \mathbf{N}_l, \qquad (1)$$

where $\mathbf{Y}_l$ is the l-th $r \times n$ received matrix, $\mathbf{H}_l$ is the $r \times t$ matrix of fading path gains, and $\mathbf{N}_l$ is an $r \times n$ matrix of noise samples.

Independent Rayleigh fading may be modeled by selecting the elements of $\mathbf{H}_l$ as unit variance complex Gaussian random variables with i.i.d. real and imaginary parts. We distinguish between *fast* fading, in which $\mathbf{H}_l$ is selected independently for each code matrix (corresponding to transmission of n space-time symbols) and *quasi-static* fading, in which $\mathbf{H}_l$ is selected independently and then held constant for groups of L code matrices (corresponding to a single packet).

III. Differential Space-Time Modulation

Hughes [28–31] proposed differential space-time codes (DSTC), which can be demodulated without channel knowledge, at a loss of 3dB in signal-to-noise ratio. DSTCs are based on unitary matrices with a group structure, forming a space-time group code. In such a code, each codeword takes the form $\mathbf{C} = \mathbf{DG}$, where $\mathbf{D}$ is a fixed $t \times n$ matrix and $\mathbf{G}$ belongs to a group of unitary matrices ($\mathbf{GG}^* = \mathbf{I}$). The n columns of $\mathbf{C}$ are transmitted as n consecutive space-time symbols.

In particular, we shall consider the following code for $t = n = 2$, with elements in the quaternary phase shift keyed (QPSK) constellation,

$$\mathcal{Q} = \left\{ \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm\begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix}, \pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \pm\begin{pmatrix} 0 & j \\ j & 0 \end{pmatrix} \right\} \qquad (2)$$

$$\mathbf{D} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad (3)$$

The group code $\mathcal{Q}$ is isomorphic to the quaternion group.

DSTCs can be differentially encoded and decoded in a way very similar to PSK. At the start of transmission, the transmitter sends the code matrix $\mathbf{C}_0 = \mathbf{D}$. Thereafter messages are differentially-encoded: to send $\mathbf{G}_l \in \mathcal{Q}$ during symbol time l the transmitter sends

$$\mathbf{C}_l = \mathbf{C}_{l-1}\mathbf{G}_l. \qquad (4)$$

The group property guarantees that $\mathbf{C}_l$ is a codeword if $\mathbf{C}_{l-1}$ is a codeword. Like differential PSK, there is a simple differential receiver [29] for $\mathbf{G}_l$ based on the two most recent blocks. This receiver computes

$$\hat{\mathbf{G}} = \max_{\mathbf{G}\in\mathcal{Q}} \Re \operatorname{tr} \mathbf{G}\mathbf{Y}_l^*\mathbf{Y}_{l-1},$$

DSTCs provide an essential building block for space-time systems that can operate with or without channel estimates at the receiver. Thus far, most work on space-time coding has assumed that perfect channel estimates are available at the receiver [9, 17, 34–36]. In certain situations, however, the channel may change too quickly to reliably estimate the channel, or may require too much overhead in terms of pilot symbols, or perhaps we want to avoid channel estimation in order to reduce the cost and complexity of the handset.

IV. Concatenated Space-Time Codes

Figure 1 shows the structure of the proposed serially concatenated code. For convenience, encoding is performed on a symbol by symbol level. The outer code $\mathcal{C}$ is a $(3, 2, \nu)$ convolutional code which outputs a single symbol from an 8-ary alphabet, which depends upon the current 4-ary input symbol as well as the last ν input symbols (a 4^ν state code). For convenience we shall consider these symbols to be members of $\mathbb{Z}_8 = \{0, 1, \ldots, 7\}$ and $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ respectively (although we do not invoke the ring axioms usually implied by this notation). We shall consider the standard maximal free distance codes [37, p. 495] with 4, 16 and 64 states (corresponding to $\nu = 1, 2$ and 3) and the natural mapping from binary to $\mathbb{Z}_4$ and $\mathbb{Z}_8$.

The stream of $\mathbb{Z}_8$ symbols are passed through a length L symbol interleaver π. The interleaved symbols are then input to the differential space-time encoder described in the previous section. The mapper simply takes each 2×2 matrix output by the inner differential code and transmits it using two consecutive space-time symbols, as described earlier.

Since the DSTC encoder is an infinite impulse response filter, it may be viewed as the "inner code" in a serially concatenated coding system. A symbol-wise turbo decoder [38, 39] for this system is shown in Figure 2. The inner *a-posteriori* decoder operates on the (fully connected) trellis of the DSTC. This is followed by de-interleaving and error control code soft-output decoding. Soft information on the space-time symbols is then fed back to the DSTC decoder, which uses it as *a-priori* information in a new decoding cycle. Such turbo decoding systems have been studied for the concatenation of standard convolutional codes with differential PSK, and very good error performance, rivaling that of Turbo codes, have been reported [40, 41].

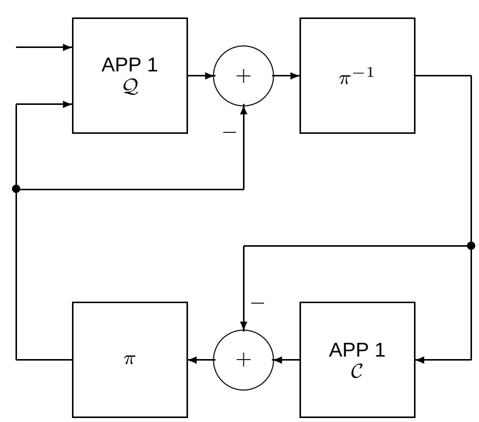

Fig. 2. Symbol-wise iterative decoder.

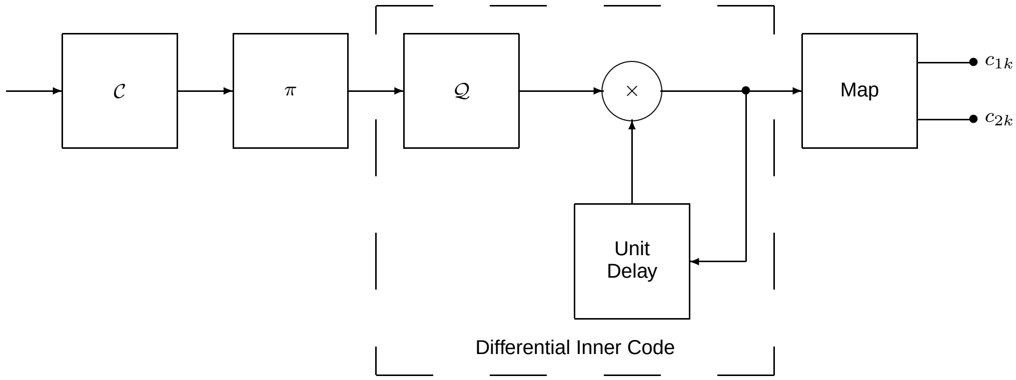

Fig. 1. Concatenation of a convolutional code with a differential space-time modulator

The inner decoder for the space-time differential code takes two inputs, the sequence of channel values $\{\mathbf{Y}_l\}$, and *a-priori* information on the transmitted symbols in the form of a sequence of probability vectors $\{\mathbf{p}_l^A\}$, where

$$\mathbf{p}_l^A = [\Pr(\mathbf{C}_l = \mathbf{G}_1), \cdots, \Pr(\mathbf{C}_l = \mathbf{G}_M)],$$

and $M = |\mathcal{Q}|$. It calculates the sequence of output *a-posteriori* probability vectors $\{\mathbf{p}_l\}$,

$$\mathbf{p}_l = [\Pr(\mathbf{G}_1|\{\mathbf{Y}_l\}), \cdots, \Pr(\mathbf{G}_M|\{\mathbf{Y}_l\})], \qquad (5)$$

using a standard implementation of the APP algorithm [42]. The *a-priori* information is removed by element-wise division. The resulting extrinsic information $\{\mathbf{p}_l^E\}$ is given as

$$\mathbf{p}_l^E = \alpha \begin{bmatrix} \mathbf{p}_l(1)/\mathbf{p}_l^A(1) \\ \vdots \\ \mathbf{p}_l(M)/\mathbf{p}_l^A(M) \end{bmatrix}; \qquad \alpha = \sum_{i=1}^{M} \mathbf{p}_l(i)/\mathbf{p}_l^A(i), \qquad (6)$$

where α is a normalization constant.

The extrinsic probabilities are de-interleaved in order to coincide with the symbols entering the outer APP decoder. This second decoder operates on the convolutional code $\mathcal{C}$. It uses $\{\mathbf{p}_l^E\}$ as *a-priori* information, and calculates new *a-posteriori* probabilities on the encoded symbols $\mathbf{G}_k$, according to the APP rule. The new extrinsic probabilities are calculated using (6). This process is iterated until a certain stopping criterion is reached.

V. EXIT Analysis

In this section we modify and apply the extrinsic information transfer analysis [43–46] to our serially concatenated system. This will give us a tool to predict the "turbo cliff" of our system and let us design and choose the convolutional outer code required for best performance.

Consider a single APP decoder. It operates on two inputs, the channel outputs $\mathbf{Y}_l$, (which may be absent if we consider the outer decoder), and *a-priori* input information $\mathbf{p}_l^A$. The decoder generates extrinsic probabilities $\mathbf{p}_l^E$ according to (5) and (6).

Let the i.i.d sequence of random variables $V_1, V_2, \ldots, V_L$ be the transmitted symbols. Under the assumption of long random interleavers, the corresponding sequence of random vectors $\mathbf{p}_l^A$ are i.i.d, as are the $\mathbf{p}_l^E$.

Define the random variables V, A and E according to $p(V, A) = p(V_l, \mathbf{p}_l^A)$ and $p(V, E) = p(V_l, \mathbf{p}_l^E)$ (note that these distributions are independent of l, due to the independence assumption). We can now define mutual informations $I(V; A)$ between the "true" symbols V and the input *a-priori* probability vectors, and $I(V; E)$ between V and the output extrinsic probability vectors. Note that this amounts to treating the prior and extrinsic probability vectors themselves as the random vectors of interest (as opposed to considering the random variables taking these vectors as their distributions). This makes sense, since it is from these vectors that the final decisions will be made. This is a new interpretation and application of this method.

Due to the non-linear operation of the APP decoder, the multi-dimensional distributions $p(V, A)$ and $p(V, E)$ are difficult to obtain analytically. We therefore estimate these measures using monte-carlo simulation of the individual codes of interest (the DSTC and the convolutional code).

The plot of $I(V, A)$ versus $I(V, E)$ is known as extrinsic information transfer (EXIT) chart. By plotting the EXIT charts for the inner and outer code on the same axes, the convergence properties of the concatenated system may be predicted, as described in [43–46].

Figure 3 shows the EXIT chart for our system using the DSTC as the inner code ($I_p = I(V, A)$ on the horizontal axis) and 4, 16 and 64 state maximal free distance rate $2/3$ convolutional codes [37, p. 495] as the outer code ($I(V, A)$ on the vertical axis). The DSTC curves are for various signal-to-noise ratios, ranging from -0.6 dB to -1.4 dB in steps of 0.2 dB. From the figure, we see that we expect the turbo threshold to occur near -0.8 dB for the 64 state outer code and around -1 dB for the 16 and 4 state codes. We also expect that the 16 and 64

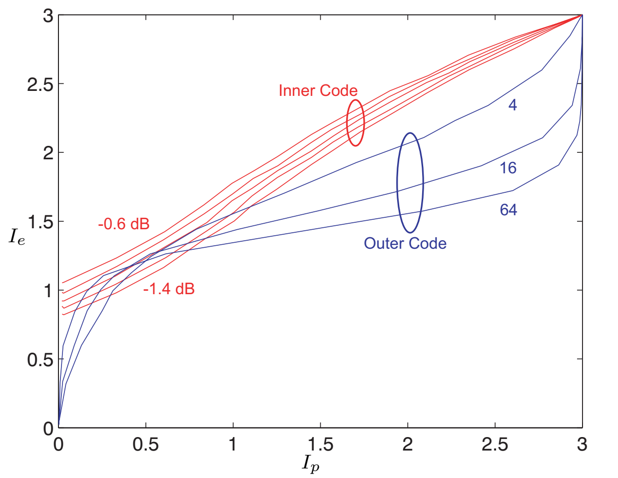

Fig. 3. Extrinsic information transfer (EXIT) chart for serial concatenation of 4,16 and 64 state convolutional codes and the differential space-time modulator.

state outer codes to result in faster convergence, since the path between the inner and outer curves are more open for these codes.

VI. Simulation Results

Figure 4 compares the performance the concatenated space-time system using a 4, 16 and 64 state, rate 2/3 maximum free distance convolutional codes [37, p.495]. The channel used is assumed to be an uncorrelated fast Rayleigh fading channel, and the decoder is furnished ideal channel side information. The interleaver length was 2048 symbols, and 25 decoding iterations were performed. As predicted by the EXIT analysis, the turbo cliff occurs near -1dB. At this code rate (1 bit per channel use), capacity is at -3.1dB [4]. Thus the concatenated coding system is achieving a bit error rate of 10^{-4} at approximately 2.3dB from capacity (for the 16 state outer code).

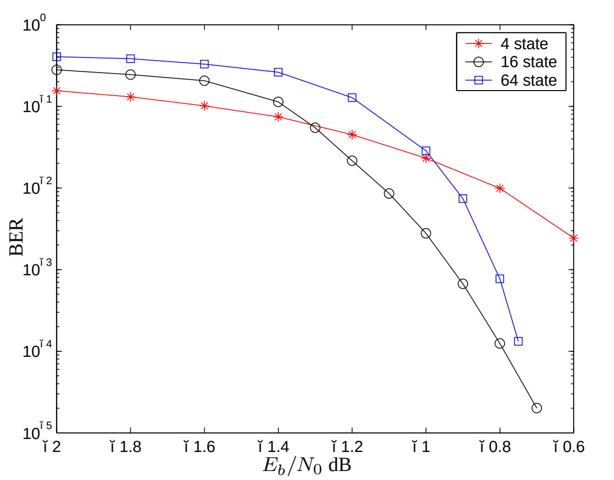

Fig. 4. Performance of the serially concatenated system compared to the rate-distortion bound of this channel.

VII. Conclusions

We have shown that the serial concatenation of standard convolutional codes with differential space-time codes can provide good performance over the multiple antenna channel, using symbol-based turbo decoding. We have further presented an EXIT chart analysis which guides the choice of the optimal component codes for a given situation and demonstrated the tightness of this analysis with example simulations. While we have used a somewhat simplistic channel model in this paper, and further assumed that ideal channel information is available to the decoder, these results are lower bounds to what can be achieved on more realistic channels where channel estimation has to be performed. These issues will be considered in a forthcoming paper, where we exploit the differential inner code to provide robustness to channel uncertainties.

References

[1] Steve Dennett, "The cdma2000 ITU-R RTT candidate submission (0.18)," Tech. Rep., TIA, July 1998.

[2] William C. Jakes, *Microwave Mobile Communications*, John Wiley and Sons, 1974.

[3] Don H. Johnson and Dan E. Dudgeon, *Array Signal Processing: Concepts and Techniques*, Prentice Hall, Englewood Cliffs, NJ, 1993.

[4] I. Emre Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585–595, Nov.-Dec. 1999, Origninally published as Tech. Memo., Bell Laboratories, Lucent Technologies, October 1995.

[5] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, March 1998.

[6] J.H. Winters, J. Salz, and G.D. Gitlin, "The capacity of wireless communication systems can be substantially increased by the use of antenna diversity," in *Conf. Inform. Sci. Syst.*, Princeton, NJ, 1992.

[7] Jack H. Winters, Jack Salz, and Richard Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Commun.*, vol. 42, no. 2/3/4, pp. 1740–1751, Feb./March/April 1994.

[8] A. Narula, M. D. Trott, and G. W. Wornell, "Performance limits of coded diversity methods for transmitter antenna arrays," *IEEE Trans. Inform. Theory*, vol. 45, no. 7, pp. 2418–2433, Nov. 1999.

[9] Siavash M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Selected Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[10] Vahid Tarokh, Hamid Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[11] Vahid Tarokh, Hamid Jafarkhani, and A. Robert Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE J. Selected Areas Commun.*, vol. 17, no. 3, pp. 451–460, March 1999.

[12] Vahid Tarokh, Nambi Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, March 1998.

[13] Ayman F. Naguib, Vahid Tarokh, Nambirajan Seshadri, and A. Robert Calderbank, "A space-time coding modem for high-data-rate wireless communications," *IEEE J. Selected Areas Commun.*, vol. 16, no. 8, pp. 1459–1478, Oct. 1998.

[14] Dakshi Agrawal, Vahid Tarokh, Ayman Naguib, and Nambi Seshadri, "Space-time coded OFDM for high data-rate wireless communication over wideband channels," in *48th IEEE Vehicular Technology Conference*, Ottawa, Canada, May 1998, vol. 3, pp. 2232–2236.

[15] Vahid Tarokh, Ayman Naguib, Nambi Seshadri, and A. R. Calderbank, "Combined array processing and space-time coding," *IEEE Trans. Inform. Theory*, vol. 45, no. 4, pp. 1121–1128, May 1999.

[16] S. Baro, G. Bauch, and A. Hansmann, "Improved codes for space-time

trellis-coded modulation," *IEEE Commun. Lett.*, vol. 4, no. 1, pp. 20–22, Jan. 2000.

[17] G.J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41–159, Aug. 1996.

[18] P.W. Wonianski, G.J. Foschini, G.D. Golden, and R.A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over rich scattering wireless channels," in *ISSSE*, 1998, pp. 295–300.

[19] H. El Gamal and A. R. Hammons, "The layered space-time architecture: A new perspective," *IEEE Trans. Inform. Theory*, 2000, Submitted.

[20] Thomas L. Marzetta and Bertrand M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 45, no. 1, pp. 139–157, Jan. 1999.

[21] Babak Hassibi and Bertrand M. Hochwald, "How much training is needed in multiple-antenna wireless links?," Technical memorandum, Bell Laboratories, Lucent Technologies, April 2000, Available at `http://mars.bell-labs.com`.

[22] L. Zheng and D. Tse, "Sphere packing in the Grassman manifold: a geometric approach to the noncoherent multi-antenna channel," in *IEEE Int. Symp. Inform. Theory*, Sorrento, Italy, June 2000, Also submitted to IEEE Trans. Inform. Theory. Pre-print available at `http://degas.eecs.berkeley.edu/~dtse/pub.html`.

[23] B. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Trans. Commun.*, , no. 12, pp. 2041–52, Dec. 2000.

[24] T. L. Marzetta, B. Hochwald, and B. Hassibi, "Space-time autocoding: Arbitrarily reliable communication in a single fading interval," in *IEEE Int. Symp. Inform. Theory*, Sorrento, Italy, June 2000, Also submitted to IEEE Trans. Inform. Theory. Pre-print available at `http://mars.bell-labs.com`.

[25] D. Agrawal, T. J. Richardson, and R. Urbanke, "Multiple-antenna signal constellations for fading channels," in *IEEE Int. Symp. Inform. Theory*, Sorrento, Italy, June 2000, Also submitted to IEEE Trans. Inform. Theory. Pre-print available at `http://mars.bell-labs.com`.

[26] Bertrand M. Hochwald and Thomas L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in Rayleigh ¤at fading," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 543–564, March 2000.

[27] Bertrand M. Hochwald, Thomas L. Marzetta, Thomas J. Richardson, Wim Sweldens, and Rüdiger Urbanke, "Systematic design of unitary space-time constellations," *IEEE Trans. Inform. Theory*, vol. 46, no. 6, pp. 1962–1973, Sept. 2000.

[28] B. Hughes, "Extensions to the theory of differential space-time modulation," in *IEEE Int. Symp. Inform. Theory*, Sorrento, Italy, June 2000, Also submitted to IEEE Trans. Inform. Theory.

[29] B. L. Hughes, "Differential space-time modulation," *IEEE Trans. Inform. Theory*, vol. 46, no. 7, pp. 2567–2578, Nov. 2000.

[30] B.L. Hughes, "Optimal space-time constellations from groups," *IEEE Trans. Inform. Theory*, 2000, Submitted.

[31] B.L. Hughes, "Space-time coding without channel estimation," in *Conf. Inform. Sci. Sys.*, Johns Hopkins University, Baltimore, MD, March 22-24 1999.

[32] B.L. Hughes, "Differential space-time modulation," in *IEEE Wireless Communications and Networking Conference*, New Orleans, LA, Sept. 22-29 1999, pp. 145–9.

[33] B.L. Hughes, "Further results on differential space-time modulation," in *IEEE Sensor Array and Multichannel Signal Processing Workshop*, Cambridge, MA, March 16-17 2000.

[34] N. Seshadri and J.H. Winters, "Two signaling schemes for improving the error performance of frequency-division-duplex (FDD) transmission systems using transmitter antenna diversity," *Int. J. Wireless Inform. Networks*, vol. 1, no. 1, 1994.

[35] A. Wittneben, "A new bandwidth ef£cient transmit antenna modulation diversity scheme for linear digital modulation," in *IEEE ICC*, 1993, pp. 1630–1634.

[36] A. Wittneben, "Base station modulation diversity for digital SIMULCAST," in *IEEE VTC*, May 1993, pp. 505–511.

[37] John G. Proakis, *Digital Communications*, McGraw–Hill, 3 edition, 1995.

[38] C Berrou, A Glavieux, and P Thitmajshima, "Near Shannon limit error-correcting coding and decoding:turbo-codes," in *IEEE Int. Conf. on Communications*, Geneva, Switzerland, May 1993, pp. 1064–1070.

[39] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: turbo-codes," *IEEE Trans. Commun.*, vol. 44, no. 10, pp. 1261–1271, Oct. 1996.

[40] P. Hoeher and J. Lodge, "Turbo DPSK: Iterative differential psk demodulation and channel decoding," *IEEE Trans. Commun.*, vol. 47, pp. 837–843, June 1999.

[41] M. Peleg, I. Sason, S. Shamai, and A. Elia, "On interleaved, differentially encoded convolutional codes," *IEEE Trans. Inform. Theory*, vol. 7, no. 7, Nov. 1999.

[42] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimising symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284–287, 1974.

[43] Stephan ten Brink, "Convergence of iterative decoding," *IEE Electron. Lett.*, vol. 35, no. 10, pp. 806–8, May 1999.

[44] Stephan ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, 2000, Submitted.

[45] Stephan ten Brink, "A rate one-half code for approaching the Shannon limit by 0.1 dB," *IEE Electron. Lett.*, vol. 36, no. 15, pp. 1293–1294, July 2000.

[46] Stephan ten Brink, "Design of serially concatenated codes based on iterative decoding convergence," in *2nd International Symposium on Turbo Codes and Related Topics*, Brest, France, 2000.